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A Model of Scrolling on Touch-Sensitive Displays

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Abstract

Scrolling interaction is a common and frequent activity allowing users to browse content that is initially off-screen. With the increasing popularity of touch-sensitive devices, gesture-based scrolling interactions (e.g., finger panning and flicking) have become an important element in our daily interaction vocabulary. However, there are currently no comprehensive user performance models for scrolling tasks on touch displays. This paper presents an empirical study of user performance in scrolling tasks on touch displays. In addition to three geometrical movement parameters — scrolling distance, display window size, and target width, we also investigate two other factors that could affect the performance, i.e., scrolling modes — panning and flicking, and feedback techniques — with and without distance feedback. We derive a quantitative model based on four formal assumptions that abstract the real-world scrolling tasks, which are drawn from the analysis and observations of user scrolling actions. The results of a control experiment reveal that our model generalizes well for direct-touch scrolling tasks, accommodating different movement parameters, scrolling modes and feedback techniques. Also, the supporting blocks of the model, the four basic assumptions and three important mathematical components, are validated by the experimental data. In-depth comparisons with existing models of similar tasks indicate that our model performs the best under different measurement criteria. Our work provides a theoretical foundation for modeling sophisticated scrolling actions, as well as offers insights into designing scrolling techniques for next-generation touch input devices.

Keywords:
Modeling, touch displays, scrolling, Fitts' law

1. Introduction

Scrolling is an important and widely used interaction technique that allows users to browse and navigate large amounts of content with limited physical screen sizes, both on classical desktop interfaces and on mobile computing devices. Essentially, scrolling moves a display window as a viewport over a larger virtual workspace to reveal regions of interest, which happens very frequently in our daily activities such as document reviewing, image editing, and map searching. For example, Byrne et al. (1999) report that users spent approximately 40 minutes on simply scrolling documents during a 5 hour web browsing session. Therefore a small improvement in the efficiency of scrolling can provide significant benefits to users.

On traditional desktops, scrolling is effective when performed by *indirect input* techniques such as rolling the mouse wheel or dragging the mouse pointer. However, as touch-sensitive devices become more popular, where gestures dominate and computer mice are typically not used, traditional scrollbars are less practical especially given extremes in display sizes (whether very large or small), as is often the case with tabletop or mobile devices (Aliakseyeu et al., 2008). Therefore the user usually approaches the target by iteratively manipulating the device with pan or flick gestures which are *direct input* techniques that mimic the familiar actions of throwing objects or shifting content. While these pan and flick gestures are similar to traditional scrolling in that they gradually reveal off-screen content linearly, they comprise a multi-step process that is not as simple as traditional aimed pointing. As such scrolling operations on touch displays become an important element in our daily interaction vocabulary, it is critical that we gain a better understanding of the mechanisms underlying how users perform such actions.

However, very few studies exist that explore the fundamentals of scrolling interactions on touch displays. Further, a comprehensive usability model has not yet been developed for them. Movement models enable researchers to improve existing user interfaces and to create novel interaction techniques. For example, the ability to predict movement times has directly led Fitts' law (Fitts, 1954) to be used for comparing and improving the efficiency of pointing devices (Soukoreff and MacKenzie, 2004). In short, current scrolling techniques on touch screens are not built upon a solid theoretical infrastructure, thus it is

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very difficult to comparatively evaluate, model, or predict human performance for the latest generation of user interfaces.

In this paper, we present an empirical study of user performance in scrolling tasks on touch displays, in which the user iteratively moves the viewport to reveal a target that is initially off-screen. Our work is the first attempt to quantitatively model scrolling interactions with direct input methods. To approach the problem, we start with an initial task analysis of scrolling interactions on touch screens in a pilot study to decompose the complicated real-world task procedure (Section 3.2). Next, based on the analysis, we formulate four mathematical assumptions for the simplified and abstract scrolling task (Section 3.3) and derive a novel quantitative model of such user interactions (Section 3.4). We then conduct a control experiment to empirically explore the effects of two scrolling interactions — panning and flicking, and two target feedback conditions — with and without distance acknowledgment (Section 4). The experimental results validate the proposed model as well as the assumptions and individual model components that support the formulation of the whole model (Section 5). In addition to the traditional R^2 measurement, we further compare our model with existing models of similar tasks under the Akaike Information Criterion (*AIC*), which represents the trade-off between model's accuracy and complexity (Akaike, 1974). Both criteria indicate that our model performs the best. Finally, we conduct comprehensive discussions of the study by further analyzing effects of the model parameters and factors and providing design implications generalized from the experiment (Section 6).

This study provides a fundamental building block for quantitatively modeling more sophisticated scrolling interactions on touch displays and offers insights into designing next-generation scrolling techniques on direct input devices. Our specific contributions include: a novel mathematical model for scrolling interactions with direct-touch input, basic observations, analysis and assumptions of the task nature in interaction modeling under multi-stage pointing paradigms, an experiment with various scrolling and content feedback techniques for validating the model and its components as well as the fundamental assumptions, thorough comparisons with the pre-existing models of similar interactions, and in-depth discussions of the model parameters, factors and its design implications.

2. Related Work

2.1. Studies on Scrolling Techniques

There exists extensive research on interaction techniques and user performance of scrolling interfaces on both traditional desktop systems and new touch-sensitive devices. Zhai et al. (1997) compared four indirect input devices in scrolling and pointing tasks of web pages browsing. They found that a mouse with an isometric joystick operated by the same hand and two hands significantly improve user performance. A very important issue related to the performance with indirect input methods is the rate of the input-output mapping, which controls the viewport movement initiated by the device movement. In particular, when the virtual workspace is very large, users easily lose either precision or speed. One solution is to dynamically change the mapping rate. For example, speed-dependent automatic zooming (Igarashi and Hinckley, 2000) and displacement-dependent auto zooming (Cockburn et al., 2005) scale the document content based on the scrolling movement parameters initiated by users. Alternatively, Cockburn et al. (2012) proposed a technique of increase scrolling performance by applying a gain function depending on the length of the document to the mouse wheel events.

For touch input devices, studies have found that panning and flicking are preferred by users to classical scrollbars (Johnson, 1995; Kaptelinin, 1995). Reetz et al. (2006) developed a flicking technique called Superflick which improves the accuracy of selecting smaller targets. But in most cases, users have to perform multiple pan or flick operations (multi-flick) to locate the target. Aliakseyeu et al. (2008) empirically compared four multi-flick techniques, and the results showed that the compound-multi-flick, which combines flicking with a displacement-based control, is preferred and as effective as the traditional scrollbar. In contrast, clutch-free panning techniques are proposed by Malacria et al. (2010) for large touch-sensitive surfaces with manipulating the parameters of a sustained oscillation. Since the screen sizes of mobile touch input devices are usually limited and users may easily get lost in a large virtual space, a number of feedback techniques have been investigated to give users acknowledgments of the target location relative to the viewport. For example, Halo (Baudisch and Rosenholtz, 2003), a off-screen target visualization technique, indicates the distance and direction of the target by drawing specific arcs on the screen. However, none of the above studies propose mathematical models to fundamentally explain user performance of various scrolling techniques. The lack of a solid theoretical framework makes it difficult to systematically extend these techniques onto different devices, compare techniques using a base model, or predict human performances in scrolling tasks.

2.2. Movement Models of Pointing and Scrolling

Fitts' law (Fitts, 1954) is widely accepted for modeling conventional rapid aimed pointing actions. Several studies have been conducted to extend Fitts' law to more complicated pointing tasks, for example, 2D and 3D target pointing (MacKenzie and Buxton, 1992; Grossman and Balakrishnan, 2004), as well as expandable targets (McGuffin and Balakrishnan, 2002). Further, Kabbash and Buxton (1995) showed that Fitts' law applies when the cursor is an area and the target is a point. Using a format similar to Fitts' law, Zhao et al. (2011) developed a model for multi-touch object manipulation. Recently, Bi et al. (2013) proposed FFitts' law to model pointing tasks with touch displays based on a dual-distribution hypothesis. While similar to one of our assumptions, they still treated the overall distribution as one single Gaussian instead of estimating the actual two "peaks" of the summed Gaussian distributions. In addition, it is an open question that if similar hypothesis holds for scrolling interaction rather than touch pointing.

For scrolling tasks, Hinckley et al. (2002) first found that Fitts' law can model certain scrolling patterns in the task of searching for specific lines in a document. But when the target is not known ahead of time in such document browsing tasks, Andersen (2005) found that Fitts' law does not hold and proposed a simple linear model under the hypothesis that maximum scroll speed is a constant. Cockburn and Gutwin (Cockburn and Gutwin, 2009) indicated similar results in their studies of examining Fitts' law and the linear model for user performance of item selection from scrolling lists. Researchers have also investigated Fitts' law based models for pointing tasks in multi-scale electronic world which includes both scrolling and zooming (Guiard and Beaudouin-lafon, 2004). However, in general, "clutching" is avoided in Fitts' law studies because it reduces the fit with empirical data (Casiez et al., 2008). This is often in contrast with the behaviors observed when users approach off-screen target with touch displays.

A very similar form of interaction to the scrolling tasks discussed above is *dynamic peephole pointing*, in which the workspace is static relative to the user and the viewport moves as a peephole over the virtual workspace. For example, one physically moves a handheld display about in 3D as a dynamic window to point and reveal virtual targets (Yee, 2003). Mehra et al. (2006) first proposed this concept to distinguish the traditional scrolling interaction, which they called *static peephole pointing*, and found significant differences between these two conditions. For such dynamic peephole pointing, Cao et al. (2008) proposed four mathematically similar models and experimentally validated the models by using pen input devices. Along the same line, Rohs and Oulasvirta (2008) investigated movement models in cases when a camera phone is used as a magic lens to locate targets physically on the background and virtually on the screen. These models divided the real tasks into a two-phase Fitts' law pointing in which the first step is to move the device to reveal the target on its display and the second step is to point the target within the display viewport.

These existing studies all utilize indirect manipulation of the scrollbars, i.e., instead of directly interacting with the display, indirect input devices such as mice and joysticks were used. No study has yet been published that models scrolling tasks with pan and flick gestures on touch displays. Further, all previous studies investigated one-stage or two-stage pointing paradigms in which each phase can be modeled by Fitts' law. These tasks are much simpler than scrolling on touch-enabled devices, where acquisition of off screen targets is achieved by panning or flicking repeatedly a (possible unknown) number of times. Therefore it is unclear whether previous models can be applied to multi-stage scrolling interaction techniques on touch displays. Although Casiez et al. (2007) proposed a multi-stage model for scrolling-like tasks using a position-rate control device, one of the assumptions was that the movement time of each stage is a constant independent with task factors, which makes it difficult to accommodate different types of devices.

Consequently, it is an open question whether user performance in scrolling tasks with direct touch input (pan and flick gestures) is similar to that when the user interacts with a conventional scrollbar using indirect input devices (mouse and joystick). In this study, we aim to develop a basic quantitative model for scrolling tasks on touch-sensitive displays. The literature (Aliakseyeu et al., 2008; Baudisch and Rosenholtz, 2003)

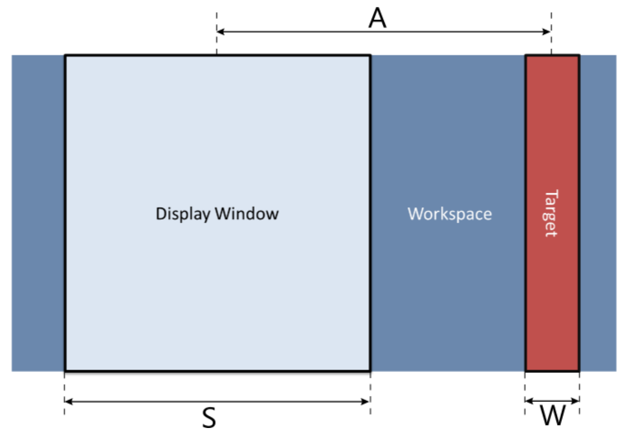


Figure 1: Target acquisition task with 1D scrolling. The user moves the background workspace to reveal the target by performing iterative operations within the display window.

indicates that the particular scrolling and feedback techniques employed can affect user performance, so our model encompasses these additional factors, including a range of scrolling techniques and distance feedback mechanisms.

3. Model Formation

3.1. Problem Description

Our focus in this study is on one-dimensional pointing tasks, which represents the first step toward developing a more generalized theoretical framework and model for scrolling on touch-sensitive displays. In this scenario, only part of a large *workspace* is exposed through a *display window*. The user may scroll the workspace by dragging the display window (in our case, in a horizontal direction) so as to reveal the target, which may then be selected (Figure 1). In keeping with the 1D (horizontal) scrolling paradigm, we have set the vertical sizes of the display and workspace to be equal to one another, so no scrolling is possible or necessary in the vertical direction.

We acknowledge that scrolling vertically is another frequent interaction in real-world scenarios, especially with mobile phones where most of the interfaces have the portrait layout. As a start, we choose to study the horizontal direction to investigate a wider physical range of scrolling interactions by making use of the horizontal layout, which would accommodate more devices with larger displays such as tablets and tabletops. We also acknowledge that true 2D scrolling may impose 2D constraints on the movement that we overlook by limiting ourselves to the 1D case. However this limitation is mitigated by the fact that 1D scrolling is employed in real-world interfaces, for example, browsing documents and viewing lists of pictures. So our model constitutes both necessary ground-work for the later development of a 2D model, and yet is also a useful model of existing systems.

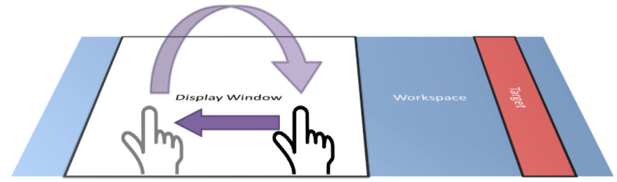
As shown in Figure 1, we employ three parameters to describe the geometry of the target acquisition task:

- **A** The distance between the center of the initial display window and the center of the target.
- **S** The size of the display window.
- **W** The width of the target.

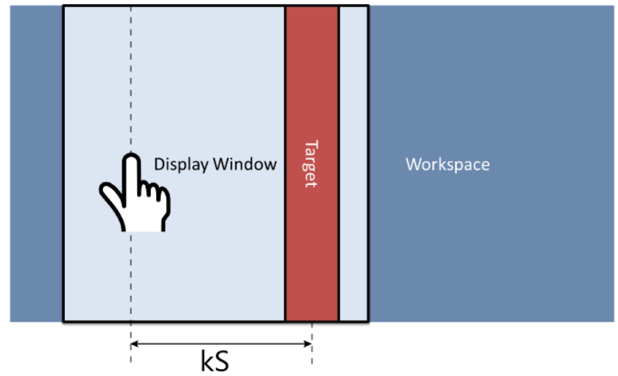
In addition, there are two other factors that could affect the performance of this pointing task.

- **Scrolling Mode** In addition to panning in which the distance and speed of the workspace movement equals that of the user's dragging action, various kinds of flicking techniques (Aliakseyeu et al., 2008) have been explored on many touch input devices. Hence, we consider two scrolling modes: *panning* and *flicking*. However, in real-world scenarios, user interactions may be a mixture of panning and flicking. In this study, we model these two scrolling modes separately as the first attempt.
- **Distance Feedback** The knowledge of how far away the target is during the searching phase may affect the user's behavior. The user may scroll the workspace at a higher speed when the distance is great, and slow down when the display window is close to the target. Such awareness of distance could be obtained by appropriate visualization techniques. Therefore we consider two feedback modes: *with distance feedback* and *without distance feedback*.

Therefore, in this study we are interested in the following four techniques which are the cross-product of the above two factors: *panning with no distance feedback* (PNF), *panning with distance feedback* (PF), *flicking with no distance feedback* (FNF), and *flicking with distance feedback* (FF).



(a) searching phase



(b) pointing phase

Figure 2: Decomposition of a 1D scrolling task on touch displays: (a) in the searching phase, the user repeatedly performs the finger-press-and-release action loop to move the workspace; and (b) in the pointing phase when the target is visible on the display window, the user points the target with his finger.

3.2. Initial Task Analysis

To model the above real-world problem, we first analyze the characteristics of the idealized scrolling task on touch displays shown in Figure 1, based on the previous work and our own informal observations with four daily touch-sensitive device users in a pilot study. The results are described in the following.

O1 *Tasks can be divided into two phases: the searching phase and the pointing phase.* As the literature indicates (Cao et al., 2008; Rohs and Oulasvirta, 2008; Casiez et al., 2007), we can divide this scrolling task into two phases: 1) the *searching phase*, in which the user iteratively moves the virtual workspace until the target emerges (Figure 2a), and 2) the *pointing phase*, which is similar to a traditional Fitts' law pointing task, and commences once the searching phase has made the target visible (Figure 2b). Similar to the basic assumption of a Fitts' law pointing task (Fitts, 1954), we assume that the user has some knowledge concerning the direction of the target, and that the intention is to reach the target as quickly and accurately as possible.

O2 *The searching phase is composed of clutches with similar properties.* In the searching phase, the target moves towards the display window a little bit in each pan or flick operation. Within each clutch, users perform the following loop operations: press the finger at some place in the display, drag it away from the direction of the target, release the finger, and repeat. Since the task is required to be done quickly, the finger-press-and-release action loop is performed continuously (without long pauses) during the searching phase. This is also concurred with the model assumptions in Casiez et al. (2007). During flicking (FNF and FF conditions) users may hold their fingers in mid-air briefly as the workspace scrolls, but this action seems still repeated rapidly and rhythmically.

O3 *The finger displacement is similar enough in clutches.* In each finger-press-and-release action loop, users tend to make rapid progress toward the target while keeping their finger movements comfortable. For simple and fast repetition, users tend to perform scrolling actions on a specific region of the display, i.e., the finger-press position and finger-release position of each loop are similar from iteration to iteration. This results in two places of interest on the screen corresponding to the average finger-press and release locations (Figure 3).

O4 *There is a ceiling effect on the overall finger displacements.* The average finger displacement demonstrates a ceiling effect due to either the physical size of the display window or maximum comfortable human wrist and arm motions. When the display width is smaller, users tend to use most of the screen area to pan or flick; whereas when it is larger, human motion limitations impose a constraint, which re-

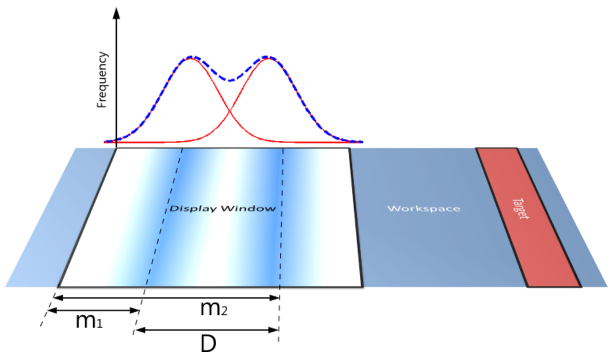


Figure 3: Distributions of finger-press and finger-release positions. Each distribution is a Gaussian (red) and the overall distribution is a mixture of two Gaussians (blue).

sults in a slower growth of the displacement with respect to the display window size.

O5 *The pointing phase has unknown starting positions.* The pointing phase is very much like a traditional Fitts' law task, except that the starting position, which approximately corresponds to the ending position of the last iteration of search phase, is unknown, especially in flicking conditions (FNF and FF) where users may overshoot because of the flicking effect. The spread of ending movement positions has been explored for similar tasks (Cao et al., 2008; Grossman and Balakrishnan, 2005). For example, a Normal distribution has been found to successfully model the spread of end movement positions in dynamic peephole pointing tasks (Cao et al., 2008).

3.3. Assumptions and Simplifications

Similar to most of the literature for modeling real-world interaction tasks, we further formalize the above task analysis into the following four tentative assumptions to abstract and simplify the complicated scrolling task, which sets the stage for the mathematical derivations of our model in the next section.

A1 Users try to comfortably maximize their finger movement displacements during the searching phase. Based on O2 and O3, we assume that the distribution of finger-press and finger-release positions in the scrolling iterations can be statistically interpreted by a mixture of two Gaussians (Bishop, ????) — a combination of two Normal distributions with different means and variances (i.e., two peaks), as shown in Figure 3. Such assumption implies the quantities m_1 and m_2, the means of the two Gaussians defined by the spread of beginning and ending locations of user dragging movements, allowing for mathematically modeling the iterated finger-press-and-release actions in the searching process (O1). Unlike Bi et al. (2013)'s dual-distribution hypothesis where they still treat the overall distribution as one single Gaussian (i.e., one peak), we want to explicitly compute the means and variances of two Gaussians to interpret the finger movement displacements.

A2 As the ceiling effects described in O4, our assumption is that the average finger displacement, $D = |m_1 - m_2|$, has a logarithmic relationship with S, i.e., $D = a + b \log_2 S$. Based on A1, the average finger displacement D can be physically interpreted by the difference of the two means from the mixture of Gaussians model. Moreover, the average motion time for each iteration has a simple linear relationship with the displacement, i.e., $T = kD + c$. The underlying intuition is that such linearity is indicated by the steering law (Accot and Zhai, 1997) for dragging the cursor across a straight tunnel and Cao et al. (2008)'s model of drawing straight line strokes (Cao and Zhai, 2007).

A3 The searching phase ending position (which is identical to the starting position of the pointing phase) is the location that the user releases their finger for the very last time in the last iteration of the searching phase, before attempting to hit the target. According to O5 and Cao et al. (2008)'s model, the assumption is that the effective pointing distance for the pointing phase is Normally distributed with the mean kS, where k is an empirically determined constant (Figure 2b).

A4 The ratio of the distance to the display window, A/S, plays a very important role in determining the number of iterations in the searching phase. Based on the analysis of the compositions of the search phase (O2) and inspirations from Casiez et al. (2007)'s model, we further assume that the number of iterations has a simple linear relationship relative to the ratio A/S.

The above assumptions may have some limitations under some circumstances since we try to formalize an ideal version of the scrolling task in practice. For example, user performance could be affected by the automatic viewport moving introduced by the flicking gestures (in FF and FNF conditions). The number of iterations might not be linear relative to the ratio A/S. Further, the movement time of each clutch iteration may not be steady. However, we argue that these assumptions may still successfully explain such behaviors in terms of the average user performance across the whole task. As the first attempt of modeling direct-touch scrolling, we aim to abstract, simplify and formalize the complicated real-world interactions, like the analysis procedure in many previous work (e.g., Cao et al., 2008; Casiez et al., 2007).

3.4. Model Derivation and Interpretation

Now we are able to derive the model grounded by the preceding analysis and assumptions. According to A1 and A2, we can represent the movement time for each iteration in the searching phase as follows,

$$\begin{aligned} T_{iteration} &= k_d \cdot D + c_d \\ &= k_d(a_d + b_d \log_2 S) + c_d \\ &= a_i + b_i \log_2 S \end{aligned} \tag{1}$$

where $a_i = k_d a_d + c_d$ and $b_i = k_d b_d$.

Combining Eq. (1) with A4, the movement time for the searching phase is

$$\begin{aligned} T_{search} &= T_{iteration} \cdot N_{iteration} \\ &= (a_i + b_i \log_2 S)\left(a_n + b_n \frac{A}{S}\right) \\ &= a_i a_n + a_i b_n \frac{A}{S} + a_n b_i \log_2 S + b_n b_i \frac{A}{S} \log_2 S \\ &= a_s + b_s \frac{A}{S} + c_s \log_2 S + d_s \frac{A}{S} \log_2 S \end{aligned} \tag{2}$$

where $a_s = a_i a_n$, $b_s = a_i b_n$, $c_s = a_n b_i$, and $d_s = b_n b_i$. We can further rewrite Eq. (2) in a more compact form since both logarithm terms evaluate over S,

$$T_{search} = a_s + b_s \frac{A}{S} + c_s \log_2 S^{\alpha_s \frac{A}{S} + 1} \tag{3}$$

where $\alpha_s = \frac{d_s}{c_s}$.

According to A3, we can model the pointing phase of this task as a conventional Fitts' law pointing task

$$T_{point} = a_p + b_p \log_2(\frac{k_p S}{W} + c) \tag{4}$$

For the sake of simplicity, we choose c to be zero, which is the same as the derivation of the dynamic peephole pointing model (Cao et al., 2008). Thus, the pointing phase can be modeled as,

$$T'_{point} = a'_p + b'_p \log_2 \frac{S}{W} \tag{5}$$

where the term k_p in Eq. (4) is subsumed into the constant a'_p using the identity $\log_2(xy) = \log_2 x + \log_2 y$.

Therefore, from Eq. (2) and Eq. (5), we can have the total movement time

$$\begin{aligned} T &= T_{search} + T'_{point} \\ &= \left(a_s + b_s \frac{A}{S} + c_s \log_2 S + d_s \frac{A}{S} \log_2 S\right) + \left(a'_p + b'_p \log_2 \frac{S}{W}\right) \end{aligned} \tag{6}$$

Next we simplify the above equation by grouping the parameters, thus obtaining the final model of the scrolling task,

$$T = a + b\frac{A}{S} + c \log_2 S + d\frac{A}{S} \log_2 S + e \log_2 \frac{S}{W} \tag{7}$$

It is interesting to note that we can further reformat Eq. (7) by grouping the logarithm terms,

$$\begin{aligned} T &= a + b\frac{A}{S} + \log_2 \frac{S^c \cdot S^{dA/S} \cdot S^e}{W^e} \\ &= a + b\frac{A}{S} + \log_2 \frac{S^{dA/S + c + e}}{W^e} \\ &= a + b\frac{A}{S} + e \log_2 \frac{S^{d/e \cdot A/S + c/e + 1}}{W} \\ &= a + b\frac{A}{S} + k \log_2 \frac{S^{\alpha A/S + \beta}}{W} \end{aligned} \tag{8}$$

where a, b, $k = e$, $\alpha = \frac{d}{e}$, and $\beta = \frac{c}{e} + 1$ are coefficients.

The formulation in Eq. (8) provides a clear separation of the constant, linear and logarithm components of the model, allowing us to apply a three-part physical interpretation to the expression with the analogy of Fitts' law and existing models as the following. Further detail explanations of the nature of the model will be discussed in Section 6.2 and 6.3.

- The term $b\frac{A}{S}$ represents the linear part of this pointing task, which is plausible since similar kind of linearity has been found in studies of scrolling tasks with other interfaces. For example, Andersen (2005) used a linear model to interpret document reading tasks using traditional scrollbars; and Casiez et al. (2007)'s model also has a linear component in a similar format.
- The last term, by analogy with Fitts' law, defines the *effective distance* of this scrolling task as $S^{\alpha A/S + \beta}$. It is intuitive that this effective distance is governed by factors A and S, and more importantly their ratio A/S plays a critical role.
- The power term $\alpha A/S + \beta$ of the effective distance explains the phenomena of repeatedly moving the workspace to reveal the target, which has a close relationship with the ratio A/S. The parameter α represents how quickly the user approaches the target, and while the parameter β represents the residual part related to the pointing phase.

4. Experiment

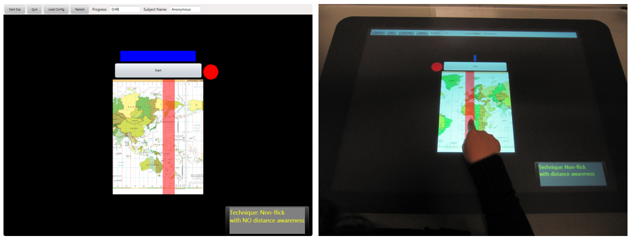
4.1. Participants

We recruited 12 right-handed volunteers (6 females), who are all university students, aged 20-28, with 0-2 years' experience using touch input devices (where 4 participants were novice touch input users). The experiment lasted for 1.5-2 hours for each participant. Participants were encouraged to take a rest between blocks.

4.2. Apparatus

The experiment was conducted on a Microsoft Surface physically arranged in the conventional configuration (i.e., flat tabletop), but raised by a height of 40 cm. This allowed our subjects to comfortably reach the entire display area when standing beside it (the lack of space for one's knees when sitting at the Surface, makes it difficult for a seated user to reach the entire display). The display has a 30 inch diagonal and a resolution of 1024×768 pixels (≈ 16.8 pixel/cm).

We chose the Microsoft Surface to utilize the physical dimensions of the display to study scrolling interactions on a wider range of viewport sizes. Hence, only a specific rectangular region of the display was revealed, allowing us to simulate smaller screen sizes corresponding to the experiment conditions (Figure 4). The widths of the viewport were selected to simulate various display window size S in 1D scrolling; the height was always 400 pixels (≈ 23.8 cm) which was large enough to impose minimal effect on the horizontal motion (Fukutoku et al., 2008). Participants used the fingers of their preferred



(a) (b)

Figure 4: Experiment setup: (a) a snapshot of the experimental software, and (b) a participant is doing the experiment.

hand for input, and they could only scroll the workspace horizontally. We used tiled world-map images as the "document" to be scrolled through, providing the participants with an awareness of movement as they scrolled.

4.3. Techniques

As discussed above, two factors have been selected for investigation: scrolling mode (panning or flicking), and feedback mode (with or without distance feedback).

We implemented the panning technique with a one-to-one mapping transfer function between the finger motion and viewport motion, which was the basic configuration in many interfaces. For the flicking technique, our implementation was similar to the multi-flick-standard (Aliakseyeu et al., 2008). That was, the scrolling speed equaled the flicking speed performed by the participant in each stroke, and the workspace continued to move at this speed if the user did not touch the surface again. The flicking speed was calculated based on the displacement and time in each stroke. Subsequent flicks enacted their respective scrolling speeds. Participants did not have knowledge about the detail implementation of the techniques. They were just instructed to approach the target as quickly and precisely as possible with panning or multi-flicking. We chose this baseline implementation because there are too many commercial touch-sensitive devices with numerous unknown parameters which are difficult to interpret, although some methods have been proposed to reverse engineer the scrolling transfer functions (Quinn et al., 2013). In our study, as the first step, we aimed to propose a baseline model as the starting point for modeling other more complex variants of flicking.

For the feedback technique, we used the length of a blue bar displayed above the viewport for indicating the relative distance to the final target (Figure 4b), since we wanted to keep the view of the core workspace itself unchanged among all the technique conditions. For each condition, the bar had the same length at the beginning; its length scaled proportionally to the current distance as progress was made toward the target, indicating a general impression about how close the target was from current location. For example, the blue bar started with the same length (Figure 4a) and its length decreased as the participant approached the target as shown in Figure 4b. While other techniques could have been applied, we chose this simpler approach because the length of the bar provided more explicit representation of the changing distance. For consistency, the blue bar was also displayed for the conditions without distance feedback but remained unchanged through the task.

4.4. Procedure

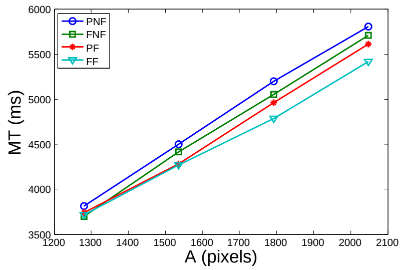
A reciprocal 1D pointing task was employed. Participants selected a target (represented as a red ribbon) in each trial, and any two successive trials were in opposing directions (to the left and then to the right and so on). The participant was asked to choose either their index or middle finger to interact with the surface, and they were required to use only this finger for input during the whole experiment. For each trial, the task was to scroll horizontally towards the target (which was initially off-screen) and successfully select it when it became visible. Participants were not allowed to scroll backwards (i.e., away from the target) until the target had appeared. Prior to each trial, participants were required to hit a "Start" button above the viewport (Figure 4a). This ensured a consistent start position of their finger and prevented the participant from making an initial unwanted gesture before scrolling. The start time was recorded from when the participant first contacted the viewport after hitting the start button. The direction of the target (left or right) was indicated by a red circle. At the end of the trial, after the target had been selected, the target disappeared and then a short beep sounded, signaling to the participant that the trial had ended.

Prior to the actual experiment, participants were given 48 practice trials, 12 for each scrolling and feedback technique combination. We recorded the movement time per trial, the number of errors, the finger contact positions and timestamps for each trial. After the experiment, we conducted a short informal interview with the participant by asking questions such as "Which technique do you like best? Why?" and "Do you think the distance feedback is helpful?"

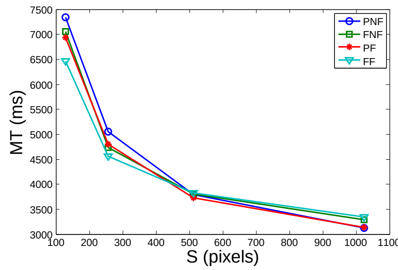
4.5. Design

For the independent variables, we used four scrolling distances: A = 1280, 1536, 1792, 2048 pixels (76.2, 91.4, 107, 122 cm), four display window widths: S = 128, 256, 512, 1024 pixels (7.62, 15.2, 30.5, 60.9 cm), three target widths: W = 16, 32, 64 pixels (0.95, 1.90, 3.81 cm), two scrolling modes: panning and flicking, and two feedback modes: with or without distance feedback. Altogether there were 192 experimental conditions. The smallest display window was about the height of the iPhone screen, and the largest display window was large enough to cover the displays of all the current commercial tablet PCs. The ratio A/S, which is in both the linear and nonlinear components of Eq. (7), plays a very important role in the model. Thus we tried to investigate a wide range of the variation in A/S (from 1.25 to 16) with the experimental conditions.

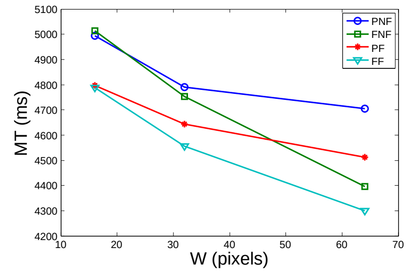
A repeated measures within-subjects design was used in the experiment. The four scrolling and feedback techniques (i.e., PF, PNF, FF, and FNF) were presented to participants with the order counterbalanced across the experiment. Under each of these techniques, participants successively selected two targets, in opposing directions (left and right), for each of the A, S, and



(a)



(b)



(c)

Figure 5: Correlations between movement times and movement factors under different techniques: panning with no distance feedback (PNF), panning with distance feedback (PF), flicking with no distance feedback (FNF), and flicking with distance feedback (FF).

W combinations presented in randomized order with 3 repetitions. In summary, for each participant, the experiment consisted of 4 *scrolling and feedback techniques* $\times$ 4 As $\times$ 4 Ss $\times$ 3 Ws $\times$ 2 *directions* $\times$ 3 *repetitions* = 1152 trials.

It is interesting to note that an alternative experimental design would be to cross variables: $S \times A/S \times W \times$ scrolling and feedback techniques[1]. Treating A and S as independent variables in the above design would not make it convenient for us to compare the performance of different techniques at similar levels of clutching iteration numbers (which are implied by A/S). Thus we could chose A/S as one of the independent variables to resolve this issue. However, this assumes a relationship between A and S, which we do not necessarily know a-priori, and it would not be easy to compare the model performance across the same A. Hence these two designs essentially complement each other rather than one necessarily being a preferred alternative. For our first attempt at validating the model, we chose the first design, similar to the original Fitts' Law experiments that were conducted in the early days of Fitts' Law research to provide initial validation. The alternative design is left for future work and may potentially offer additional and more complete verification of the model.

5. Results

5.1. Movement Times and Errors

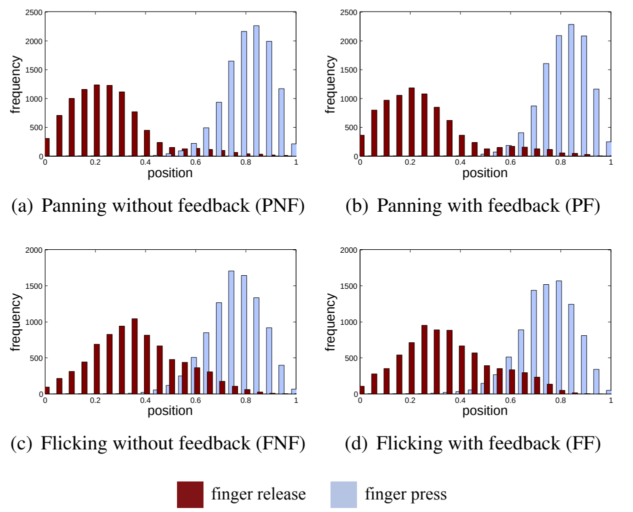
The average of movement time T was 4687 ms for this scrolling task. A repeated measure ANOVA test showed significant effects for A ($F_{3,33} = 561.9, p < .001$), S ($F_{3,33} = 186.13, p < .001$), and W ($F_{2,22} = 67.72, p < .001$) on T. Pairwise means comparisons also indicated that T increases monotonically as A increases and T decreases monotonically as S and W increases (Figure 5). There was no significant effect for either scrolling mode nor feedback mode on T. But there were significant interactions for $S \times$*scrolling mode* ($F_{3,33} = 12.41, p < .001$), $S \times$*feedback mode* ($F_{3,33} = 5.212, p < .005$) and $A \times$*scroll mode* ($F_{3,33} = 6.813, p < .002$). However, it worths noting that these significant interactions might be because of the non-overlapping A/S values for different S's in our experimental design. Further empirical observations are needed to make the conclusions firmly.

Figure 5 shows the correlations between the movement times and the three movement factors under the four scrolling techniques. In general, the movement times of techniques without distance feedback (PNF and FNF) seem to be slower than those with distance feedback (PF and FF), indicating distance acknowledgment could improve the user performance under certain conditions. Several interesting findings can be identified from these charts as follows.

First, in Figure 5a, we can see that the advantages of flicking and distance feedback tend to be negligible when A is small; whereas when A grows larger, FF results in the fastest technique, which is plausible because flicking and distance feedback benefit more to the user for larger A. Second, as Figure 5b shows, for smaller S, the average movement time of flicking techniques (FNF and FF) is much less than that of panning techniques (PNF and PF); but for larger S, the difference tends to be much smaller and panning are even faster for the largest S. This may be because participants were likely to utilize more screen width when told only simple panning was available, which resulted in the viewport traveling a little faster than that in flicking. However, this could also due to the fact that there existed large differences of iteration numbers between various A's because of our experimental design. Future additional experiments as described in Section 4.5 would provide more concrete supports. Third, Figure 5c indicates that the increasing of W has larger influence (i.e., deeper slope) on movement times of flicking techniques (FNF and FF). The reason could be that the marginal easiness of pointing larger targets is more significant with flicking techniques because it makes the viewport moving at certain speed in the pointing phase of the whole task.

An error was counted when the participants tapped outside the target. However, especially with techniques using flicking, the participant could also tap to stop workspace from moving. Thus in this case we measured an error only when the target was present inside the display window and the participant tapped within a certain distance (10 pixels, ≈ 0.6 cm) beyond the edge of the target, otherwise it was recorded as a tap to stop the scrolling workspace. The average error rate was 3.49%. As expected, there was a significant effect for W ($F_{2,22} = 116.1, p < .001$) on error rate, and pairwise means comparison shows that

[1] We thank the reviewers for pointing this out.



(a) Panning without feedback (PNF)
(b) Panning with feedback (PF)
(c) Flicking without feedback (FNF)
(d) Flicking with feedback (FF)

Figure 6: Histograms of the finger-press (blue) and finger-release (red) positions on the display window. The target is on the right. The position data was normalized (divided by S) and was flipped for conditions where the targets were on the left, before inclusion in this graph.

the error rates for $W = 16$ pixels (6.64%) are much higher but error rates are about the same for $W = 32$ pixels (2.08%) and $W = 64$ pixels (1.76%). There was also a significant effect for scrolling mode ($F_{1,11} = 301.4, p = 0.002$) in which flicking conditions (FNF and FF) have higher error rates (5.34%) than panning conditions (PNF and PF) which were 1.65%.

5.2. *Assumption Verifications*

The four assumptions introduced in Section 3.3 are the foundations of our model derivation. In this section, we revisit these assumptions by verifying them with the empirical data gathered from the experiment.

5.2.1. *Finger-press and Finger-release Positions (A1)*

The histograms for normalized finger pressing and releasing positions on the display window for the four techniques are shown in Figure 6. We clearly observe that the overall distribution of the hitting positions has two peaks, which can be interpreted with the mixture of two Gaussians model (Bishop, ????), supporting A1. Moreover, for each technique, the one-sample Kolmogorov-Smirnov test rejected the null hypothesis that those hitting positions are drawn from a normal distribution ($p < .001$), indicating that our two Gaussion mixture model assumption is necessary to interpret the data and Bi et al. (2013)'s dual-distribution hypothesis does not apply here since they still assume the overall combined distribution is a single Gaussian (i.e., one peak).

The results of ANOVA tests indicated that S has a significant effect on these hitting positions ($F_{3,33} = 23.47, p < .001$). Further, the distances between the means of Gaussians is smaller for the flicking modes, which implies the participants could achieve their desired scrolling speed with less effort and smaller displacements.

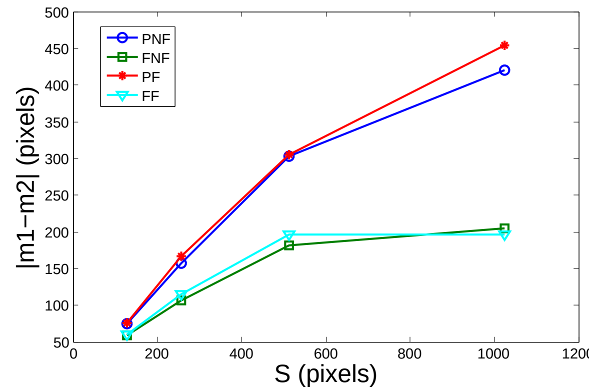


Figure 7: Correlations between $|m_1 - m_2|$ and S under four techniques: panning with no distance feedback (PNF), panning with distance feedback (PF), flicking with no distance feedback (FNF), and flicking with distance feedback (FF).

5.2.2. *Finger Displacements (A2)*

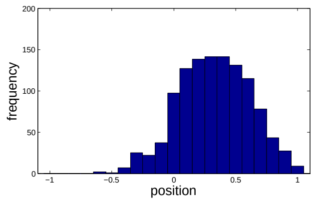
We also investigated the average finger displacement for each iteration which is represented as the distance between the two means of Gaussians, $|m_1 - m_2|$. In each technique, to compute the means, we ran the expectation-maximization (EM) algorithm (Bishop, ????) to fit a mixture of two Gaussians with the hitting positions for each display window width S. The EM technique is an iterative method for finding maximum likelihood estimates of parameters in statistical models, which are means and variances of the two separated Gaussians here.

The trend of $|m_1 - m_2|$ growing with S is shown in Figure 7. We regressed the data using function $y = a + b \log x$, which yielded $R^2 > 0.98$ for all techniques, supporting A2 and also confirming the ceiling effect of view size in Guiard and Beaudouin-lafon (2004)'s study. In addition, there was a significant main effect for S ($p < .001$) on the movement time of each iteration under all techniques.

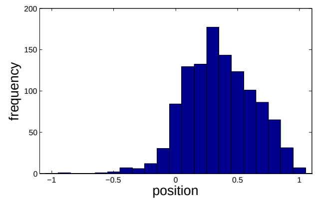
5.2.3. *Searching Phase Ending Positions (A3)*

Though several studies have indicated that the spread of ending positions follows a Normal distribution for similar actions (Cao et al., 2008; Grossman and Balakrishnan, 2005), results in Figure 8 provide further support for A3 in this touch display scrolling task, especially for panning techniques (PNF and PF). For flicking techniques, the last finger-release position could be several screens away because of the movement initiated by the flicking gestures.

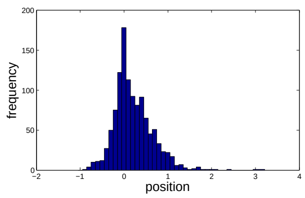
Moreover, an ANOVA indicated that S ($F_{3,33} = 20.48, p < .001$), W ($F_{2,22} = 285.2, p < .001$), scrolling mode ($F_{1,11} = 16.35, p = 0.0019$), and feedback mode ($F_{1,11} = 17.24, p = 0.0016$) have statistically significant effects on the spread. Note that the means of the ending positions tend to skew in the direction of the user's movement approaching the target. Also, the spread of ending points for the flicking actions reveal long tails trailing away in the opposite direction. Neither of these features resemble a Normal distribution. Thus further experiment is warranted.



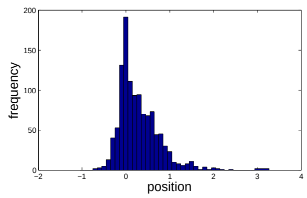
(a) Panning without feedback (PNF)



(b) Panning with feedback (PF)

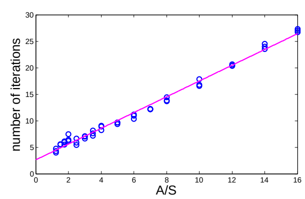


(c) Flicking without feedback (FNF)

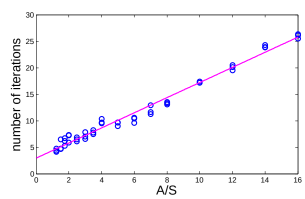


(d) Flicking with feedback (FF)

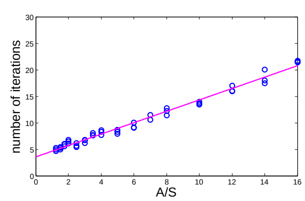
Figure 8: Histograms of the searching phase ending positions (last finger-release positions before hitting the target). The target is at position 0 and the participant approaches target from the right. The position data was normalized (divided by S) and was flipped for conditions where the targets were on the left, before inclusion in this graph.



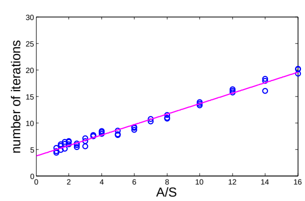
(a) Panning without feedback (PNF)



(b) Panning with feedback (PF)



(c) Flicking without feedback (FNF)



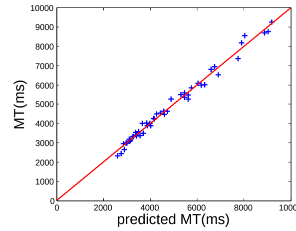
(d) Flicking with feedback (FF)

Figure 9: Linear fitting results of the number of iterations with respect to the ratio A/S.

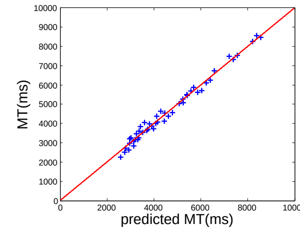
5.2.4. Iteration Numbers (A4)

In order to evaluate A4, we investigated the relationship between the number of iterations and the ratio of the distance to the display window width, A/S. Figure 9 shows that this relationship is highly linear yielding $R^2 > 0.96$, which gives a strong justification for A4.

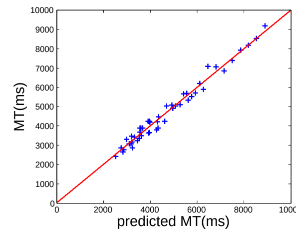
As expected, there were significant effects for A ($F_{3,33} = 289.7, p < .001$) and S ($F_{3,33} = 148.8, p < .001$). Furthermore, scrolling mode had a significant effect ($F_{1,11} = 10.33, p = 0.0082$) and pairwise comparison showed that flicking modes had a smaller number of iterations. We can also observe this from the slope of the fitted lines in Figure 9, in which the ones of flicking techniques have slower increasing rates with respect to the ratio A/S.



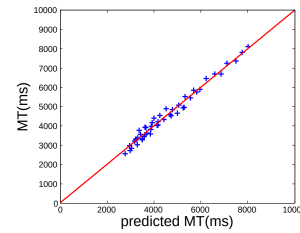
(a) Panning without feedback (PNF)



(b) Panning with feedback (PF)



(c) Flicking without feedback (FNF)



(d) Flicking with feedback (FF)

Figure 10: Correlations between predicted movement times and observed movement times.

5.3. Model Fittings

Because of the nonlinearity of the proposed model (Eq. (8)), we fit it with the experimental data by turning the process into a nonlinear optimization problem with the objective function — sum of squared errors between predicted and observed movement times. We used function `fminsearch` [2] in MATLAB, which finds the minimum of a scalar function of several variables starting at an initial estimate by using a derivative-free method. Since this highly nonlinear object function may have multiple local minimas, we ran the fitting process for ten times and took the average values of the solutions in the end. For each technique, the raw data points were collapsed to experimental conditions, i.e., $4\ As \times 4\ Ss \times 3\ Ws = 48$ data points to be fitted in each model, by averaging the times for each condition. Table 1 shows the estimated parameters and R^2 values, indicating that our model fits the experimental data very well ($R^2 \geq 0.97$). Figure 10 further illustrates the goodness of fit between the empirical and the predicted movement times.

In addition to the four mathematical assumptions, which are verified in the preceding sections, our model is based on three fundamental components that model the movement times in each clutch iteration, the searching phase, and the pointing phase (see Eq.(1), Eq.(3), and Eq.(5) respectively). To further examine the model, we extracted the experimental data to perform regression analysis of the three component models individually. For the fitting of $T_{iteration}$ in Eq.(1), we took the average clutch iteration times in each geometry factor condition (A, S, and W) of the four techniques. For the other two component

[2] http://www.mathworks.com/help/techdoc/ref/fminsearch.html

Techniques	a (ms)	b (ms)	k (ms/bit)	α	β	R^2
PNF	2.36	0.002	175	0.395	1.65	0.98
PF	-1.36	0.003	172	0.370	1.72	0.98
FNF	-5.36	0.002	243	0.264	1.39	0.97
FF	-6.54	0.001	271	0.211	1.39	0.97

Table 1: Model fitting results under different techniques

Model component	PNF	PF	FNF	FF
Eq (1): $T_{iteration} = a + b\log_2 S$	0.98	0.96	0.87	0.92
Eq (3): $T_{search} = a + b\frac{A}{S} + c\log_2 S^{\alpha\frac{A}{S}+1}$	0.97	0.98	0.97	0.98
Eq (5): $T_{point} = a + b\log_2 \frac{S}{W}$	0.94	0.85	0.86	0.94

Table 2: Model fitting results (R^2 values) for different components.

models (T_{search} and T_{point}), we used similar approach of fitting the entire model above, but with the corresponding movement times recorded in the experiment. Table 2 shows the fitting results, indicating that all three model components explain the experimental data very well with relative high R^2 values, which further assures the validity of our model. Detail information about the fitted coefficients can be found in Table A.6.

Moreover, we ran similar model fitting procedures for the longer format of the model in Eq.(7) and its corresponding searching phase in Eq. (2), since we were curious about whether different model formulations would affect the estimation of coefficients in this non-linear fitting process. The results showed that we encountered large negative values for coefficients a and b in the longer model formats, which seemed not very plausible or practical. Details of the coefficient values can be found in Table A.7. We suspect that the differences might due to the complicated parameter interactions during the iterative fitting process in two model formats, causing the solution searching towards very different local minimas. However, future studies with more empirical data are warranted. Thus, we suggest using the compact model format (Eq.(7)) in practice, and if needed, one can transform the fitted parameters to those in the long format through the formulations described in Section 3.4.

5.4. Model Comparisons

This scrolling task on touch displays is similar enough to the traditional scrolling and the dynamic peephole pointing task that it is worth comparing our results with the models proposed for these interactions. Of particular interest, the previous models are the two-part Fitts' law models for dynamic peephole pointing with a stylus (Cao et al., 2008) and a camera phone (Rohs and Oulasvirta, 2008), the model for multi-scale navigation using joysticks (Guiard and Beaudouin-lafon, 2004), and the clutch model for pointing with rate-control devices (Casiez et al., 2007).

We compared our model with the models mentioned above. Note that Cao et al. (2008) proposed many similar models; for this comparison we compared our model against the best performing of these models. For the multi-scale model (Guiard and Beaudouin-lafon, 2004), we added a constant parameter a to the equation, allowing this model to achieve a better fit with our data, because scale was not varied in our experiment. In Casiez et al. (2007)'s model, we set the CD gain as 1 (because we used direct touch for interaction) and the constant parameter T_c as the average clutch time of the corresponding technique. Results in Table 3 show that our model yields the highest R^2 values for all techniques, indicating that our model fits the experimental data the best. Further details about the coefficients of the model fittings can be found in Table A.8 in Appendix A.

However, evaluating competing models via R^2 values alone may introduce a bias for models with different numbers of parameters, especially for non-linear models. As the number of parameters grows, a model's descriptive ability will be improved but the stability of estimations of parameters decreases which may cause the model's predictive ability decreases (Ren et al., 2005). The goal of statistical modeling is to select a model with strong predictive ability, but the traditional evaluation method, R^2, which indicates the goodness of fit to the observed data, cannot represent the predictive ability of the model. A better approach for model evaluation is the Akaike Information Criterion (AIC) developed by statistical model selection, which describes the trade-off between model's accuracy, i.e., the descriptive ability, and model's complexity, i.e., the stability of parameter estimations (Akaike, 1974). AIC is defined by the maximum log-likelihood and the number of parameters to be estimated. Based on the assumption that the model errors are Normal independent and identically distributed (i.i.d.), the formula is

$$AIC = 2k + N\ln\frac{\sum\epsilon_i^2}{N} \tag{9}$$

where k is the number of parameters, N is the number of data points, and ϵ_i is the estimated residuals from the fitted model (Burnham and Anderson, 2004).

The model with the smallest AIC value can be regarded as the best one and only the difference of AIC values has meaning that is used for model selection. Denote AIC_i is the AIC value of the ith model candidate and AIC_{min} is the minimum of those, in practice, the quantity $\exp(\frac{AIC_{min}-AIC_i}{2})$ can be interpreted as the relative probability that the ith model, which measures how probable other model as the best model minimizes the informa-

Model	Goodness	PNF	PF	FNF	FF
Two-part Fitts' law I (Cao et al., 2008)	R^2	0.90	0.90	0.87	0.88
$T = a + b(n\log_2(\frac{A}{S}+1) + (1-n)\log_2(\frac{A}{W}+1))$	AIC	613	606	618	598
Two-part Fitts' law II (Rohs and Oulasvirta, 2008)	R^2	0.91	0.90	0.89	0.89
$T = a + b\log_2(\frac{A}{S}+1) + c\log_2(\frac{S}{2W}+1)$	AIC	609	601	612	593
Multi-scale model (Guiard and Beaudouin-lafon, 2004)	R^2	0.82	0.81	0.81	0.80
$T = a + \frac{k}{S}\log_2(\frac{A}{W}+1)$	AIC	641	634	636	618
Clutch model (Casiez et al., 2007)	R^2	0.80	0.91	0.92	0.84
$T = 2T_c\lfloor\frac{A}{cS}\rfloor + a + b\log_2(\frac{A-cS\lfloor\frac{A}{cS}\rfloor}{W}+1)$	AIC	651	602	593	613
Our model	R^2	0.98	0.98	0.97	0.97
$T = a + b\frac{A}{S} + k\log_2\frac{S^{\alpha A/S+\beta}}{W}$	AIC	546	540	559	533

Table 3: Comparisons of models under two criteria.

tion loss (Burnham and Anderson, ????). As Table 3 shows, our model yielded the smallest AIC values with the minimum difference $\Delta AIC = 64$ across all the techniques and all the models, which means the relative probability of other models is no larger than e^{-26} that is a fairly small number. Thus our model outperforms the others under this evaluation method as well.

5.5. Informal Interview Results

We conducted a short informal interview following the experiment to quantify the participants' subjective opinions of the four scrolling and feedback conditions. We expected flicking with distance feedback to be the most preferred technique. However, only 7 of the 12 participants regarded it as their preferred method and most of these (6 out of the 7) had at least one year of experience using touch input devices such as iPhone and iTouch. For novice participants with no experience, 3 out of 4 preferred the panning techniques. They found the flicking hard to control and they sometimes overshot the display window. Ten participants felt that the distance feedback was very helpful especially when S is small and the other two felt that it was only somewhat helpful. All participants with prior experience of using touch-input reported that they frequently use the gestures employed in the experiment for daily scrolling tasks.

6. Discussion

6.1. Model Comparison In-Depth

While the performance of our model is the best under these two criteria, one may wonder whether that the two-part Fitts' law models (Cao et al., 2008; Rohs and Oulasvirta, 2008) or the clutch model (Casiez et al., 2007) might be adequate when precision is not highly required, because they generated around 0.9 R^2 values under some techniques. However, we believe that these models may not truly represent the nature of this scrolling task. It is a question that if the two successive phases of the task (searching phase and pointing phase) can be explained as the model's state, because the 0.9 R^2 values may due to the interactions of different parts of the model and its parameters. The pointing phase is very similar to a traditional pointing task modeled by Fitts' law (Fitts, 1954), which has been validated in many literatures and has similar mathematical formats in all of these models including ours. In the case of modeling scrolling tasks on touch screens, the searching phase dominates the movement time of the whole task (78% on average from our experimental data). So it is critical for a model to reflect the task nature by well interpreting the searching phase with its corresponding component.

Hence, we conducted similar comparison procedures in Section 5.4 by fitting the corresponding model components to the times of the searching phase only. After eliminating the second pointing phase part, both the two-part Fitts' law models (Cao et al., 2008; Rohs and Oulasvirta, 2008) are reduced to Fitts' law, and the clutch model (Casiez et al., 2007) turns into a linear form. The searching phase of our model, i.e., Eq. (3), is already verified with the experimental data in Section 5.3. As Table 4 shows, the other two models have large decreases in performances, in contrast our model still yields very high R^2 values. Detail results of the parameter values are shown in Table A.9 in Appendix A. Therefore, dynamic peephole pointing and clutching with rate-control devices are different tasks in nature from this direct-touch scrolling task concerned in this paper. It further indicates that we need different models for those tasks, and our model well explains the nature of this scrolling tasks so that it can provide valuable implications in practice.

6.2. Model and Parameters

From the fitting results in Table 1, we may tentatively interpret the meaning of the estimated parameters in terms of the underlying physical meaning of the static peephole pointing task in the multi-stage scrolling paradigm. The effective distance of this scrolling task, $S^{\alpha A/S+\beta}$, is affected by parameters α and β. In particular, the parameter α describes the nature of the technique where smaller values typically indicate faster techniques. As shown in Table 1, the values of α for flicking techniques are relatively smaller than for panning techniques. In addition, the parameter β, which represents the residual part of the repeated movement, has smaller values for flicking techniques as well, which indicates that the pointing phase of the flicking action

Model	Goodness	PNF	PF	FNF	FF
Two-part Fitts' law (searching phase)	R^2	0.82	0.82	0.80	0.82
$T_{search} = a + b\log_2(\frac{A}{S}+1)$	AIC	610	604	614	592
Clutch model (searching phase)	R^2	0.59	0.50	0.54	0.35
$T_{search} = 2T_c\lfloor\frac{A}{cS}\rfloor$	AIC	692	694	689	692
Our model (searching phase)	R^2	0.97	0.98	0.97	0.98
$T_{search} = a + b\frac{A}{S} + c\log_2 S^{\alpha\frac{A}{S}+1}$	AIC	540	552	557	552

Table 4: Comparisons of model fittings to the searching phase of the whole scrolling task.

Model	Goodness	PNF	PF	FNF	FF
Our model I	R^2	0.98	0.98	0.97	0.97
$T = a + b\frac{A}{S} + k\log_2\frac{S^{\alpha A/S+\beta}}{W}$	AIC	546	540	559	533
Our model II	R^2	0.96	0.97	0.97	0.96
$T = a + k\log_2\frac{S^{\alpha A/S+\beta}}{W}$	AIC	568	550	556	542
Our model I (searching phase)	R^2	0.97	0.98	0.97	0.98
$T_{search} = a + b\frac{A}{S} + k\log_2(S^{\alpha A/S})$	AIC	540	552	557	552
Our model II (searching phase)	R^2	0.98	0.98	0.97	0.97
$T_{search} = a + k\log_2(S^{\alpha A/S})$	AIC	550	538	555	538

Table 5: Comparisons of the new model and the original model for both the whole scrolling task and only the searching phase.

contributes less. This may be because the continued sliding of the workspace followed by each flick operation, to some extent, reduces the effort for pointing at the target.

Parameters b and k represent the weights of the linear and nonlinear components respectively. From Table 1, we can observe that the relative ratio k/b is larger for flicking techniques, indicating the nonlinear part contributes more under such condition. Moreover, what surprised us is that all of the values of b are relatively small which implies that the linear part of the model, $b\frac{A}{S}$, has very small impact upon the whole model. This might be because that the nonlinear component whose expression includes the ratio A/S has incorporated most of the linear aspects of the scrolling interaction. Thus we may consider tentatively removing this term to yield a new simpler formulation of the model

$$T = a + k\log_2\frac{S^{\alpha A/S+\beta}}{W} \tag{10}$$

As Table 5 shows, by evaluating this new model in a similar approach as we did in the previous section, we found that it achieves about the same performance as the original model, which yielded similar R^2 and AIC values in the regression analysis of both the whole task and the searching phase. Therefore this version of the model may be preferred in practice because it has fewer parameters.

6.3. Model and Factors

Since most of the time necessary to complete this scrolling task is spent on the searching phase, in which the factors A and S play a more important role, we choose to study how these two factors affect the new model. According to Eq. (10), the movement time T, if viewed as a function of $S \in (0,\infty)$, has a single maxima point which is determined by α, β, and A (Figure 11). But Figure 5 shows that the time T decreases when S increases for each technique. This implies that all of the sampled points in this experiment fall in the region beyond the maxima point towards infinity. However, whether only this region is practical for real data and tasks is still open for study.

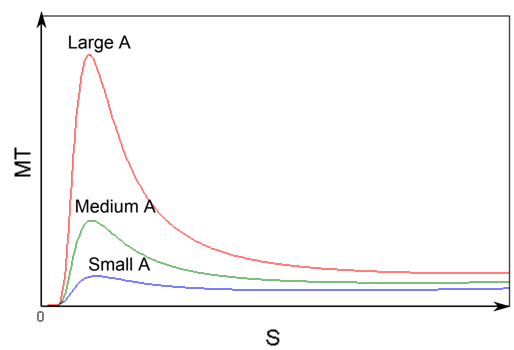


Figure 11: The relationship between movement time with parameters A and S.

Further, as shown in Figure 11, when the target distance A is larger, the movement time drops down significantly as the display window width S increases, whereas for smaller values of A, the slope tends to change slowly. Therefore the model indicates that the factor S affects the time less when A is smaller, because fewer iterations are needed to reveal the target, and so a larger window does not have a significant impact on the whole pointing task for smaller distances. Another interesting observation that we can make from the shape of the model is that when S is really large the curves tend to be flat, which means the window width does not affect the movement time very much when the window is extremely large. An extreme condition occurs when the target is initially inside the display windows, in this case time is not affected at all by the window width S.

6.4. Design Implications

From the experiment and the analysis of the results, we have observed several factors that are likely to be of interest to interaction designers.

First, from the spread of Gaussians in Figure 6, we find that the participant is more varied at the end of scrolling and more consistent at the beginning, because the variance for finger-press positions (blue) is smaller than that for finger-release positions (red). Hence, especially in flicking modes, designers may consider adding tolerance to these loose ending positions. For example, assign the same scrolling speed for strokes within a specific range of lengths. Otherwise, the speed may change abruptly for each operation if only simple mapping functions are employed. This approach may improve the smoothness of viewing the content during scrolling.

Second, it worths noting that the finger displacement remains relatively constant around 200 pixels ($\approx$ 12 cm) in flicking modes for screen sizes equal to or larger than $S = 512$ pixels ($\approx$ 30 cm), as shown in Figure 7. This distance could be the desirable human flicking distance. The participants tended to use more space for panning. Hence, for large display windows, the length of the stroke may indicate the user's intention - quick exploration (flicking) or carefully browsing (panning).Therefore hybrid scrolling techniques that combine panning and flicking dynamically, adjusted to the behavior of the user, may be a good choice for new designs.

Third, it is very interesting that there is a strong linear relationship between A/S and the number of iterations (Figure 9). Therefore, techniques making this line flatter, which means less number of iterations at certain A/S, could significantly speed up the target acquisition. From the parameter fittings of the model (Table 1), we can see that such relationship is reflected by the α parameter. Thus this parameter, coupled with of the slope of the A/S versus the number of iterations curve, could be one of the ways to evaluate, compare and further refine the scrolling techniques.

6.5. Limitations

As our work is the first attempt to model user performance for direct-touch scrolling under a multi-stage pointing paradigm, there are several limitations in our current study which we aim to address.

First, we notice that it is a fact that the flicking implementation of most commercial devices has the friction simulation. However, it is difficult to select a baseline and representative implementation to model because every device may have a different friction mechanism. And there is current no study indicating which implementation is the best. Of the techniques in our study, panning, in which the distance of the background movement exactly equals users' finger displacement, is a special case of flicking where the flicking friction is infinite; and the multi-flick-standard, in which the speed of the content after flicking is the same as the flicking speed imposed by users, is a kind of flicking with zero friction. Because real-world implementations lie in between these two extreme conditions, we believe that our model is very likely to predict the movement time under various types of flicking techniques.

Similar for feedback techniques, in a real application, a user may be able to get at least some information regarding the target location, from off-screen visualization techniques, e.g., Halo (Baudisch and Rosenholtz, 2003), or the user's familiarity with the content or presentation methods of targets, e.g., address books listed in alphabetical order and documents with an outline or bookmark window. However, we also deal with two extreme conditions for distance feedback: no distance feedback and complete distance indication. In our experiment, participants had prior knowledge of the location of the target through two means: direction indicated before scrolling and distance feedback during scrolling. Thus, we believe that practical applications likely fall between the two target information extremes we modeled, which were already verified in the experiment.

Third, in this paper, we only studied the horizontal direction of the 1D scrolling. Although on smaller devices such as the iPhone, vertical scrolling seems to be more common. We selected the horizontal setup as the original Fitts' study (Fitts, 1954). Also, such configuration allows us to investigate larger range of the display window sizes on the Microsoft Surface. Specifically, horizontal and vertical scrolling motion shares a similar nature, in which a linear and repetitive multi-stage physical action is employed to accomplish motion toward the target. Moreover, the model would likely be applicable to linear 2D peephole pointing as well, so long as a similar interaction framework was present. The effect of direction, i.e., angle, upon 2D scrolling is left for future work.

Last, there might be some differences in user behaviors between our specific experimental configuration (the Microsoft Surface) and other touch-sensitive devices in our everyday life, such as mobile phones and tablets. The way that the user holds the devices, view angles, and display orientations may affect the scrolling manipulations, which is different from the user standing by the tabletop to perform scrolling interactions off-hand. Thus, future studies on such mobile devices are warranted to further validate the model.

In short, our goal here is to provide a baseline model that could be extended to model more complicated flicking gestures and scrolling tasks, like that the original Fitts' law (Fitts, 1954) which first models the basic pointing and then is extended to the area cursor cases (Kabbash and Buxton, 1995) and multi-dimensional pointing (MacKenzie and Buxton, 1992; Grossman and Balakrishnan, 2004).

7. Conclusion and Future Work

In this paper, we presented an empirical study of scrolling tasks on a touch-sensitive display. Based on four assumptions drawn from the observation and analysis of user behaviors, we proposed a quantitative model for the simplified and formalized scrolling tasks. Then, an experiment was conducted to validate this model under different scrolling and feedback techniques. All of the assumptions have been supported by analyzing the experimental data. Moreover, regression analysis indicated that our model as well as its three critical components fit the data very well. We also compared our model to existing models under two evaluation methods: traditional R^2 measures and AIC. The results showed that our model outperformed the pre-existing models in many situations. Based on the fitting results and discussions of the model factors, we proposed a new model with fewer parameters but the same predictive performance. Finally, we conducted in-depth discussions in many aspects, including interpretations of the physical meanings of model parameters and factors, a set of design guidelines generalized from the experiment, and limitations of the current study.

As for future work, we first aim to address the issues discussed in the limitations of this study. We plan to test our model with other interaction techniques, further assuring its validity in a wider application domains in the real-world. For example, we would like to evaluate our model under other feedback techniques such as speed-dependent auto-zooming (Igarashi and Hinckley, 2000), and other scrolling techniques and friction models (Aliakseyeu et al., 2008). Particularly, we aim to conduct more experiments, such as the alternative design described in Section 4.5, to further validate some of the observations we had and ensure the robustness of the model. We are also interested in studying the user performance of acquiring 2D targets with 2D display windows involving multi-touch interaction techniques such as zooming and rotating. Further, we plan to experiment this model on mobile devices or wall-size displays and with other input interfaces such as the stylus or the mouse. Another interesting point to look at is that users may have time delays in performing scrolling interactions due to mental processing or choice reaction, which are not considered in our current model. Thus a GOMS or KLM like model (Card et al., 1980, 1983) could be constructed based on our formulation to capture the various procedures in the user's complex scrolling and navigation behaviors.

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Appendix A. Complete Model Fitting Results

Iteration	Technique	a (ms)	b (ms/bit)			R^2
$T_{iteration} = a + b \log_2 S$	PNF	-690	148			0.98
	PF	-644	140			0.96
	FNF	-301		107		0.87
	FF	-310		106		0.92
Searching phase	**Technique**	**a (ms)**	**b (ms)**	**c (ms/bit)**	**α**	**R^2 R^2**
$T_{search} = a + b\frac{A}{S} + c \log_2 S^{\alpha \frac{A}{S} + 1}$	PNF	915	0.001	93.8	0.557	0.97
	PF	950	0.004	92.0	0.658	0.98
	FNF	770	0.002	153	0.393	0.97
	FF	865	0.007	142	0.466	0.98
Pointing phase	**Technique**	**a (ms)**	**b (ms/bit)**			**R^2**
$T_{point} = a + b \log_2 \frac{S}{W}$	PNF	285	166			0.94
	PF	350	148			0.85
	FNF	386	210			0.86
	FF	373	230			0.94

Table A.6: Fitting results of the three basic models components under different techniques.

Long form model	Technique	a (ms)	b (ms)	c (ms/bit)	d (ms/bit)	e (ms/bit)	R^2
$T = a + b\frac{A}{S} + c \log_2 S + d\frac{A}{S} \log_2 S + e \log_2 \frac{S}{W}$	PNF	-3078	-470	383	148	144	0.98
	PF	-2939	-429	383	137	143	0.98
	FNF	-3192	-325	412	127	309	0.98
	FF	-2558	-312	339	111	244	0.97
Long form model in searching phase	**Technique**	**a (ms)**	**b (ms)**	**c (ms/bit)**	**d (ms/bit)**		**R^2 R^2**
$T_{search} = a + b\frac{A}{S} + c \log_2 S + d\frac{A}{S} \log_2 S$	PNF	-3120	-486	343	151		0.97
	PF	-3095	-488	348	146		0.98
	FNF	-4575	-411	485	139		0.97
	FF	-2656	-354	316	116		0.98

Table A.7: Fitting results of the long forms of the model under different techniques.

Our model I	Technique	a (ms)	b (ms)	k (ms/bit)	α	β	R^2	AIC
$T = a + b\frac{A}{S} + k\log_2\frac{S^{\alpha A/S+\beta}}{W}$	PNF	2.36	0.002	175	0.395	1.65	0.98	546
	PF	-1.36	0.003	172	0.370	1.72	0.98	540
	FNF	-5.36	0.002	243	0.264	1.39	0.97	559
	FF	-6.54	0.001	271	0.211	1.39	0.97	533
Our model II	Technique	a (ms)	k (ms)	α (ms/bit)	β		R^2	AIC
$T = a + k\log_2\frac{S^{\alpha A/S+\beta}}{W}$	PNF	2.48	176	0.411	1.67		0.96	568
	PF	-2.36	173	0.382	1.71		0.97	550
	FNF	-6.34	242	0.268	1.39		0.97	556
	FF	-5.52	269	0.223	1.38		0.96	542
Two-part Fitts' law I (Cao et al., 2008)	Technique	a (ms)	b (ms/bit)	n			R^2	AIC
$T = a + b(n\log_2(\frac{A}{S}+1) + (1-n)\log_2(\frac{A}{W}+1))$	PNF	-1357	2104	0.872			0.90	613
	PF	-1049	1918	0.862			0.90	606
	FNF	-1908	2069	0.789			0.87	618
	FF	-890	1702	0.792			0.88	598
Two-part Fitts' law II (Rohs and Oulasvirta, 2008)	Technique	a (ms)	b (ms/bit)	c (ms/bit)			R^2	AIC
$T = a + b\log_2(\frac{A}{S}+1) + c\log_2(\frac{S}{2W}+1)$	PNF	-1864	2219	379			0.91	609
	PF	-1548	2030	373			0.90	601
	FNF	-2530	2218	577			0.89	612
	FF	-1378	1820	466			0.89	593
Multi-scale model (Guiard and Beaudouin-lafon, 2004)	Technique	a (ms)	k (ms·pixels/bit)				R^2	AIC
$T = a + \frac{k}{S}\log_2(\frac{A}{W}+1)$	PNF	2606	10623				0.82	641
	PF	2647	95780				0.81	634
	FNF	2671	97939				0.81	636
	FF	2865	80453				0.80	618
Clutch model (Casiez et al., 2007)	Technique	a (ms)	b (ms/bit)	c	T_c (ms)		R^2 R^2	AIC AIC
$T = 2T_c\lfloor\frac{A}{cS}\rfloor + a + b\log_2(\frac{A-cS\lfloor A/cS\rfloor}{W}+1)$	PNF	396	454	1.962	565		0.80	651
	PF	476	373	2.286	551		0.91	602
	FNF	256	542	2.268	602		0.92	593
	FF	387	505	2.399	588		0.84	613

Table A.8: Fitting results of our model and its alternative form as well as comparisons with other models under different techniques using R^2 and AIC.

Our model I	Technique	a (ms)	b (ms)	c (ms/bit)	α	R^2	AIC
$T_{search} = a + b\frac{A}{S} + c\log_2 S^{\alpha\frac{A}{S}+1}$	PNF	915	0.001	93.8	0.557	0.97	540
	PF	950	0.004	92.0	0.658	0.98	552
	FNF	770	0.002	153	0.393	0.97	557
	FF	865	0.007	142	0.466	0.98	552
Our model II	**Technique**	**a (ms)**	**k (ms)**	**α (ms/bit)**		**R^2**	**AIC**
$T_{search} = a + k\log_2(S^{\alpha A/S})$	PNF	865	69.0	0.955		0.98	550
	PF	950	87.7	0.690		0.98	538
	FNF	770	228	0.264		0.97	555
	FF	915	125	0.418		0.97	538
Two-part Fitts' law	**Technique**	**a (ms)**	**b (ms/bit)**			**R^2**	**AIC**
$T_{search} = a + b\log_2(\frac{A}{S} + 1)$	PNF	915	0.001			0.82	610
	PF	950	0.004			0.82	604
	FNF	770	0.002			0.80	614
	FF	865	0.007			0.82	592
Clutch model	**Technique**	**c**	**T_c (ms)**			**R^2**	**AIC**
$T_{search} = 2T_c\lfloor\frac{A}{cS}\rfloor$	PNF	1.781	565			0.59	692
	PF	1.815	551			0.50	694
	FNF	1.941	602			0.54	689
	FF	2.316	588			0.35	692

Table A.9: Fitting results of our model and its alternative form in the searching phase as well as comparisons with other similar models under different techniques using R^2 and AIC.